Problem
Solving File

Appendix E:
PSFile

Part 1: Teacher's Notes

Part 2: Recording Sheets

Part 3: The Problem Cards

Part 1: Teacher's Notes

Introduction to the File

Problem solving has always been of major importance in mathematics; in recent years it has become the central goal of mathematics education in North America.

"Mathematics As Problem Solving" is "Standard 1." National Council Teachers of Mathematics (1989)

More locally, Alberta Education has interpreted problem solving to be:
1. learning about how to solve problems
2. learning concepts through problem solving
3. applying the concepts learned to new problem situations
4. creating problems that exemplify the concepts learned

The first two aspects are integral to the MathWorlds approach, and are developed within the ongoing storyline of each module setting.

The fourth aspect is the focus of the Problem Book which is a place for students to create problems, and work on those created by other students.

The third aspect is the focus of The PS-file. Therefore, the PS-file should be taken to be:
- complementary to the program as a whole
- necessary to ensure a complete program

The Polya Problem Solving Model

The Polya Problem Solving Model is now a standard feature in school mathematics:
- **Understanding** the problem
- **Planning** investigative action
- **Doing** the investigation
- **Looking Back** upon the report of the investigation

If as a detective you were to carefully examine the MathWorlds Program looking for evidence of the Polya Model you may be surprised to find that it was used:
- to organize the **Table of Contents**
- to format the **Section Summaries**
- to organize the **Lesson Plans**
- to organize the student inquiry process
- to organize mathematical problem solving activities in each module.
- to structure the *Program Notes*
- to help students learn how to solve problems
- to help students learn concepts through problem solving

Problem Solving Objectives

The PS-file is designed to provide opportunities for students to explicitly work on the problem solving objectives targetted for each grade by Alberta Education. Surprisingly, there is only one new objective for grade five:

P-19 collects and uses data - line graphs

Because only one problem solving objective is new for Level 7, the problems selected for this file focus on objectives set for grade four:

P-11 knows what information is implied

P-12 knows what information is missing

P-13 knows what information is extraneous

P-24 works problem backwards

P-31 alters the problem and finds the effect

P-32 generalizes the solution

P-33 creates problems that exemplify the concepts learned

It must be noted that these objectives are all interrelated: It makes little sense to be working on P-11, "knows what information is implied" in isolation of P-12, "knows what information is missing" or P-13, "knows what information is extraneous". Therefore, in any problem there are several objectives operating simultaneously. To realize this is to acknowledge that problem solving itself is an integrative force. This is particularly true for the attitudinal objectives:

P-1 demonstrates willingness to find a solution to a problem

P-2 perseveres in finding a solution to a problem

P-3 demonstrates flexibility in finding solutions to problems

P-4 presents ideas clearly

P-5 works both independently and in a group situation

These objectivs are the focus of MathWorlds generally as well as the PS-file in particular.

File Contents and Organization

The PS-file contains:

Student Problem Cards. The file provides 18 problems, each presented on a card.

Blank Problem Cards. These blank cards are for additional problems that you may wish to include in the PS-file. The Student Problem Book would be an excellent source of good problems.

Student Report Sheets. A special report sheet is provided on a CopyMaster for use by students to report their work on each problem. Information on the form will help teachers to systematically monitor, file and record student work.

Student Summary Sheet. Students may use this sheet to keep an on-going record of work done in the file. It will also provide teachers with a bird's-eye view of each student's progress.

Class Summary Sheet. Teachers may use this summary form to create a class profile.

Notes for Teachers. For each problem the following information is provided:

- an ID number and title for the problem
- the problem solving objective(s) involved
- the content strand objective(s) involved
- comments about the solution(s)

Managing the File

Accessing the File

A convenient way to use the file is to have it available for easy student access at any time during the day.

Operational Suggestions for the Student

- Students select a Problem Card, and a Report Sheet.
- Work on the problem during spare time
- Store their on-going work in their folder
- Return the Problem Card when they no longer need to access it
- Submit the Report Sheet to the teacher when complete
- Note their progress in the Student Summary Sheet

Operational Suggestions for the Teacher

- Open a PS-file folder for each student. Students can place their Report Sheets in this folder when they complete a problem
- Keep each student's Summary Sheet (which is filled in by the student) in the folder as well
- Occasionally, but regularly, peruse the Student Report Sheets in order to provide feedback and to become aware of possible opportune times to access DMP for comprehensive diagnosis and instruction
- on occasion hold a conference with students to discuss some of their work

Integrating with the *Problem Book*

- Each student's Problem Book will be a valuable and rich source of interesting and relevant problems for other students
- Select problems from student Problem Books for inclusion in the PS-file. Blank cards are provided for this purpose. It would be a great way to celebrate and honor the work of your students
- Research indicates that student-generated problems capture the interest of other students and serve as a wonderful way to tailor the PS-file to your class.

Problem 1

Dr. Tan the Detective

Problem solving objectives

P-11   knows what information is implied

P-12   knows what information is missing

P-24   works problem backwards
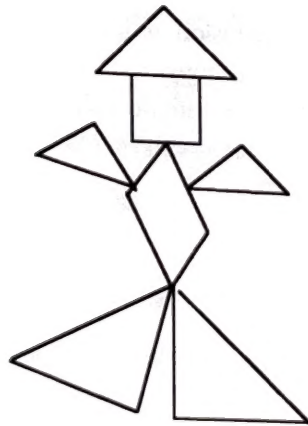
P-31   alters the problem and finds the effect

Content strand objectives

M-27   estimates and measures mass

M-28   mass in g, kg and tonnes (t)

M-45   Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

Solution Commentary

A strategy in solving this case could be as follows: The sketch must be of Detective Tan if it can be used as an outline to make Detective Tan from Tangrams.

Supporting evidence would be to make make Detective Tan showing all the tangram pieces in the construction as shown here.

Tan's mass is 64 kg; one leg is 16 kg

Students could make similar problems in their Problem Book

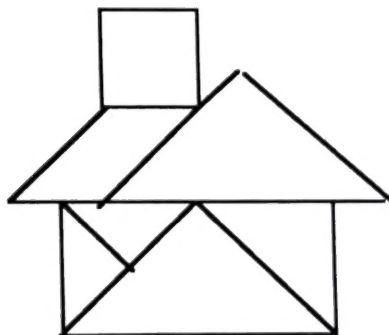# Problem 2

The House of Tan

Problem solving objectives

P-11 knows what information is implied

P-12 knows what information is missing

P-24 works problem backwards

P-31 alters the problem and finds the effect

Content strand objectives

M-27 estimates and measures mass

M-28 mass in g, kg and tonnes (t)

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

Solution Commentary

Supportive evidence could include showing that the house can be made of tan-grams. Of course other houses could be too. (See the strategy comments for Problem 1.)

House mass: 16 t

The chimney is rather large.

No windows

Any other astute observation

Problem 3
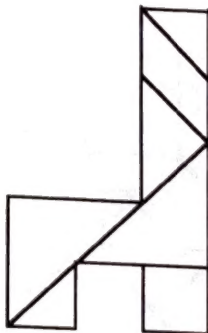
The Throne of Dr. Tan

Problem solving objectives

P-11 knows what information is implied

P-12 knows what information is missing

P-24 works problem backwards

P-31 alters the problem and finds the effect

Content strand objectives

M-27 estimates and measures mass

M-28 mass in g, kg and tonnes (t)

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

Solution Commentary

The throne can be made from Tangrams as shown. (See the strategy comments for Problem 1.)

The throne would weigh 160 kg which is very likely too heavy even for two strong men to cart around. But with the use of a dolly they could manage.

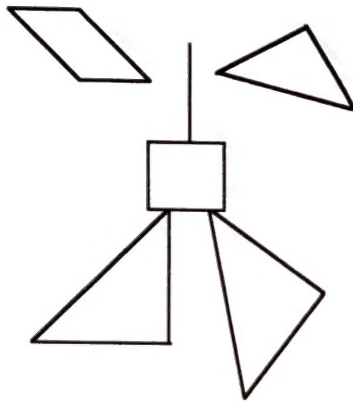# Problem 4

Tan's Partner

Problem solving objectives

P-11 knows what information is implied

P-12 knows what information is missing

P-24 works problem backwards

P-31 alters the problem and finds the effect

Content strand objectives

M-27 estimates and measures mass

M-28 mass in g, kg and tonnes (t)

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

Solution Commentary

Sketch of Tan's partner can be made of tangrams as shown. (See the strategy comments for Problem 1)

48 kg

Legs a bit heavy (but so are Tan's!)

Super hearing capability

Head is symmetrical

Problem 5
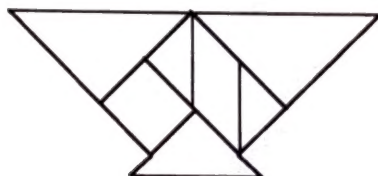
The Golden Rice Bowl

Problem solving objectives

P-11 knows what information is implied

P-12 knows what information is missing

P-24 works problem backwards

P-31 alters the problem and finds the effect

Content strand objectives

M-27 estimates and measures mass

M-28 mass in g, kg and tonnes (t)

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

Solution Commentary

Here is one way of constructing the bowl. (See the strategy comments for Problem 1)

80 g

Problem 6

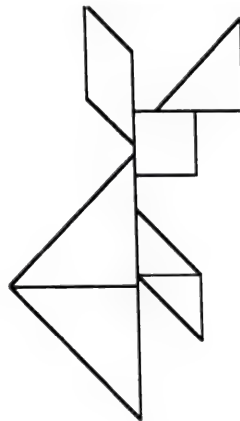## A Mouse In the House of Tan

Problem solving objectives

P-11   knows what information is implied

P-12   knows what information is missing

P-24   works problem backwards

P-31   alters the problem and finds the effect

Content strand objectives

M-27   estimates and measures mass

M-28   mass in g, kg and tonnes (t)

M-45   Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

Solution Commentary

Here's one version of the mouse. (See the strategy comments for Problem 1)

200 g

Problem 7

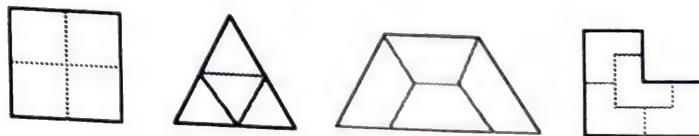## Detective Faraday to the Rescue

Problem solving objectives
P-31 Alters the problem and finds the effect

Content Strand Objective
M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

G-13 Constructs two-dimensional figures

Solution Commentary

Students could make up some other shapes that could also be cut into congruent quarters.

Problem 8

SleepyHead's Rude Awakening

Problem solving objectives

P-11 knows what information is implied

P-12 knows what information is missing

P-24 works problem backwards

P-31 alters the problem and finds the effect

Content strand objectives

M-8 time to seconds

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

Solution Commentary

TriHead had a poor excuse since through cosmetic surgery he can be shown to exactly cover the square.

Problem 9

Sending Enough Rectangles

Problem solving objectives

P-22 Guesses and Checks

Content strand objectives

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

M-46 Demonstrates understanding of the relationship between area and perimeter

Solution Commentary

1. 5 units
2. 6 units

Problem 10

Posthole Pete Did It

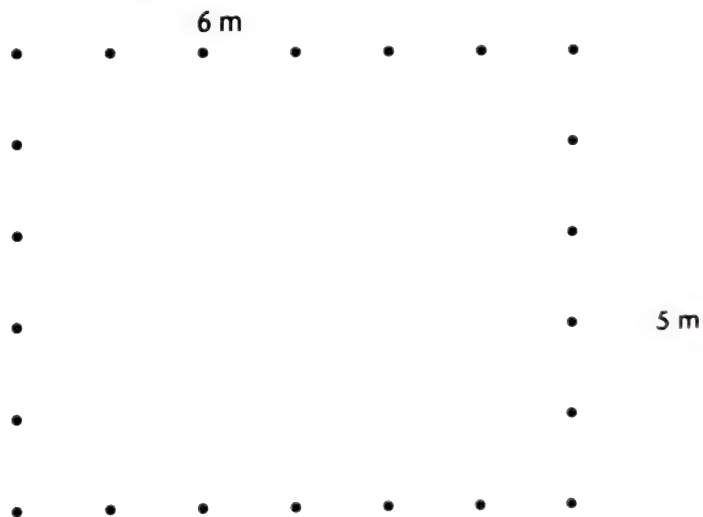### Problem solving objectives

P-1   knows what information is implied

P-12   knows what information is missing

P-13   knows what information is extraneous

P-31   alters the problem and finds the effect

Content strand objectives

M-17   Appropriate units of length

M-44   Finds and compares perimeter of polygons

M-45   Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

M-46   Demonstrates understanding of the relationship between area and perimeter G-7. Classifies and names 2-D figures as pentagons, hexagons, octagons

Solution Commentary

As the diagram shows, Posthole Pete's garden was 5 m by 6 m. He needed 22 posts so why would he steal only 20? And because Pete was an excellent student of mathematics, he wouldn't make mistakes like this.

Problem 11

Weighty Evidence

Problem solving objectives

P-1   knows what information is implied

P-12 knows what information is missing

P-13 knows what information is extraneous

Content strand objectives

M-8   time to seconds

M-28 mass in g, kg and tonnes (t)

Solution Commentary

None of the girls are likely guilty as they all weigh 80 kg and therefore were not the ones who stood on the scale just after midnight.

If they are guilty it is for some other crime — like eating too much!

Problem 12

Time Out for DeciDays

Problem solving objectives

P-1 knows what information is implied

P-12 knows what information is missing

P-13 knows what information is extraneous

P-32 generalizes the solution

Content strand objectives

M-8 time to seconds

Solution Commentary

Manfred's unit	Days	Equivalent to hours	minutes	seconds
day	1	24	00	00
deciday	0.1	02	24	00
centiday	0.01	00	14	24
milliday	0.001	00	01	26.4
decaday	10	240	00	00

Class period (about 40-50 minutes) would be about 30-35 millidays or 3 centidays.

Recess (10-15 minutes) would be about 7-10 millidays.

Christmas vacation would be a decaday.

Question: Would a "holiday" fit into the scheme?

Problem 13

Capturing Pentagons on the Geoboard

Problem solving objectives

P-1 knows what information is implied

P-12 knows what information is missing

Content strand objectives

M-44 Finds and compares perimeter of polygons

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

M-46 Demonstrates understanding of the relationship between area and perimeter

G-7 Classifies and names 2-D figures as pentagons, hexagons, octagons

Solution Commentary

The Largest Pentagon on the Geoboard would look like this (a square with one corner cut off):

Measurements:

- Perimeter: 15.4 units (students will have to estimate this)
- Area: 5.5 square units

Problem 14

Capturing Hexagons on the Geoboard

Problem solving objectives

P-1 knows what information is implied

P-12 knows what information is missing

Content strand objectives
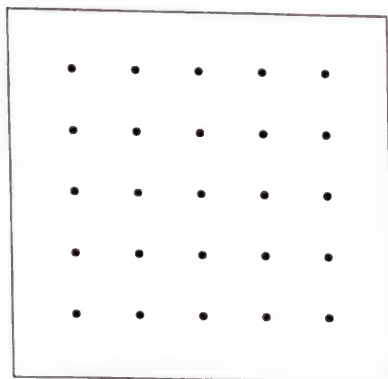
M-44 Finds and compares perimeter of polygons

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

M-46 Demonstrates understanding of the relationship between area and perimeter

G-7 Classifies and names 2-D figures as pentagons, hexagons, octagons

Solution Commentary

The Largest Hexagon on the Geoboard (a square with two vertices cut off):

Measurements:

- Area: 15 square units
- Perimeter: Approximately 14.8 units (students can use estimation

Problem 15

Capturing Octagons on the Geoboard

Problem solving objectives

P-1   knows what information is implied

P-12 knows what information is missing

Content strand objectives

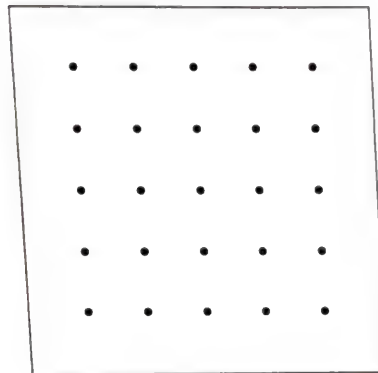**M-44** Finds and compares perimeter of polygons

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

M-46 Demonstrates understanding of the relationship between area and perimeter

G-7   Classifies and names 2-D figures as pentagons, hexagons, octagons

Solution Commentary

The Largest Octagon on the Geoboard (a square with four vertices cut off):

Measurements:

- Area: 14 square units
- Perimeter: 13.6 units
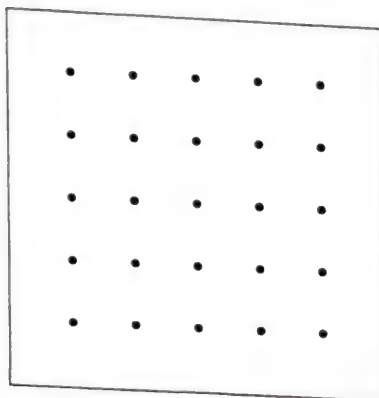
Problem 16

Search and Find

Problem solving objectives

P-1 knows what information is implied

P-12 knows what information is missing

Content strand objectives

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas

M-46 Demonstrates understanding of the relationship between area and perimeter

Solution Commentary

9 squares of this size   4 squares of this size   1 square of this size

Problem 17

Capture Through Measurements

Problem solving objectives

P-12 knows what information is missing

P-24 works problem backwards

Content strand objectives

M-44 Finds and compares perimeter of polygons

M-45 Uses manipulatives and diagrams in a problem-solving situation to find and compare areas
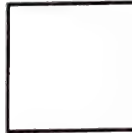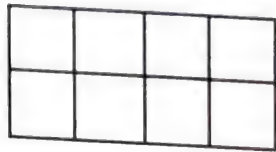
M-46 Demonstrates understanding of the relationship between area and perimeter

Solution Commentary

A = 8 square units.   P = 12   A = 8 square units.   P = 14 units

Problem 18

Recording a Bath

Problem solving objectives
P-19 Collects and uses data — line graphs

Content strand objectives
D-13 Constructs line graphs
D-16 Distinguishes among the types of graphs and discusses reasons for using each type of graph

Solution Commentary
Solutions will vary considerably. Here is a sample story:

> Detective Aromato couldn't wait to get the water started in the tub.
>
> At 8:15 he turned the hot water on full blast and started to shed his smelly clothes.
>
> Just before 8:17 he put his big toe into the water and yelled *"Yelp! That's hot"* and at
>
> 8:17 turned the cold water on full blast too.
>
> One minute later, at 8:18, he turned both taps off, stuck his big toe in again and it felt so good. He soaked himself thoroughly for five full minutes, feeling warm and relaxed.
>
> At 8:23 he pulled the plug, and gurgle, gurgle, gurgle, he went down the drain with the water! And boy did he smell bad again.
>
> (The ending is optional.)

Part 2: Recording Sheets

Student Report Sheet

Problem Number _____   Investigator _____

Date Completed _____   License _____
mm / da / h / min

Problem Title _____

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Student Summary Sheet

Name ———————————

Problem ID	Date started	Date completed	Partners (if any)	Observations/Comments
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				

Class Summary Sheet

Problems Solved

Student Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

PSFile

Part 3: The Problem Cards

Problem 1

Dr. Tan the Detective

The Case

The simple sketch shown here was made of Detective Tan as outlined in the dimness of a street lamp.

Your task is to show that this likely is a sketch of Detective Tan.

Provide evidence to support your case.

Needed Descriptive Information

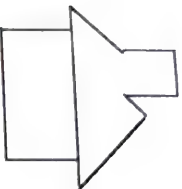The mass of Tan's left arm is 4 kg. What is Tan's mass? How heavy is one of his legs?

Problem 2

The House of Tan

The Case
Police believe that this is a sketch of Tan's house. Could you supply some supportive evidence for this belief?

Needed Descriptive Information
If the chimney has a mass of 2000 kg, how many tonnes is the House of Tan? Anything unusual about Tan's house?

Problem 3

The Throne of Dr. Tan

The Case

Dr. Tan always wanted to be a drummer in a rock band. Rock drummers have a special name for the stool they sit on. They call it a **throne**. Dr. Tan had a special throne made for himself. Could this be a sketch of it? Support your case with a more detailed sketch.

Needed Descriptive Information

If the mass of the rear leg of the throne is 20 kg, could Dr. Tan carry his throne with him to play different gigs?

Problem 4

Tan's Partner

The Case

Detective Tan was often seen with a partner. Could this be a sketch of his partner?

Needed Descriptive Information

If the head of Tan's partner has a mass of 3 kg, what is his/her total mass? Is Tan's partner a male or female? Anything unusual about him/her?
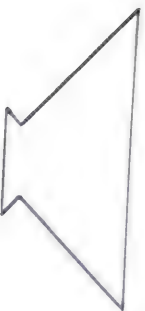
Problem 5

The Golden Rice Bowl

The Case

Is this a shot of a rice bowl that Detective Tam might use? It is known that Tam's bowl has a triangular base.

Support your case with further detail of the bowl's construction

Needed Descriptive Information

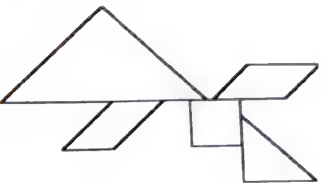If the bowl weighs 640 grams, what is the mass of the base of the bowl?

Problem 6

A Mouse in the House of Tan

The Case

A rather large mouse has been terrorizing Tan's house the past few weeks. Could this be the mouse of the House of Tan?

Needed Descriptive Information

If the body of the mouse is 800 grams, what is the mass of his rear ear?

Problem 7

Detective Faraday to the Rescue

To his great surprise, when Detective Faraday dropped in to interview Mr. Smithers, a birthday party was in full swing. Four children, including Avia, Mr. Smither's daughter, were having a merry time. Just as Faraday arrived, the cake-cutting ceremony was about to begin.

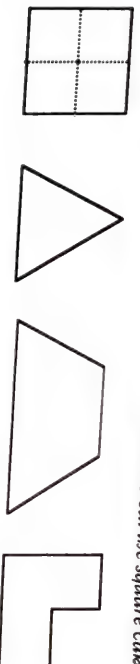Surprisingly, there were four cakes, each with a different shape.

Mr. Smithers: "I thought each of you would like your very own cake!"

The four children then began arguing over who should get which cake.

Mr. Smithers: "I've got an idea. What if each of you got one quarter of each cake. Does that seem fair?"

Avia: "A great idea! And I think each of our pieces should be the same shape as each of the big cakes. Then we would each have a miniature cake just like the big cakes."

Mr. Smithers: "You mean like this?" And he proceeded to cut the square cake.

"Yes," said Avia, "That's exactly it. Now how about the others?"

Mr. Smithers knife hovered over the other cakes for several moments, but he couldn't see how to cut them.

Mr. Smithers: "Faraday, my good man: Good detective that you are, could you cut these cakes properly?"

Faraday, ever thankful for his training to see in different ways, began cutting the cakes with authority and precision. How did he cut the other three cakes?

Problem 8

SleepyHead's Rude Awakening

Special Note: Solve Case Set 3-3 (Cosmetic Surgery) and Case Set 3-9 (CoverUp Caper) before trying this problem

Very early in the morning, 1999 04 20 06 00 to be precise,

FlatHead

was rudely awakened and ordered to do a cover-up job on a poster...

... that was stapled to a wooden power pole.

TriHead wanted to sleep in. "I'm much too big for the poster and shouldn't be wasted on such a small job," he groaned. "That poster is only 4 square centimetres and I'm bigger."

Did TriHead have a good excuse?

The cosmetic surgery lab was put on alert.

Problem 9

Sending Enough Rectangles

Have you completed **Case Set 3-9 CoverUp Caper**?

How many rectangles like this...

... should be sent to do cover-up jobs on a funny-looking poster like this?

(horizontal body with a big head in here)

(horizontal body with big stomach and big head in here)

Problem 10

Posthole Pete Did It

Posthole Pete was known as a shady character. He rarely had any money, and when he did he didn't have it for long. But he liked vegetables (he said carrots helped him to see better at night) and he also liked mathematics. He once got 100% on a math test when he was in grade 5.

He planted a garden that was rectangular in shape, but dogs and other creatures kept digging it up. He needed a fence. He wanted a strong fence, one with a post every meter. His garden was 5 meters wide and had an area of 30 square meters. Pete understood these kinds of things quite well.

But poor Posthole Pete had no money. And no one would lend him any.

One day Muskrat Lumber reported a robbery — 20 posts were stolen. The investigating officer noted in his notepad: "4 other posts left lying in the yard — why weren't these taken too?"

The police came immediately to pick up Posthole Pete. Everyone knew he needed a fence and didn't have the posts.

Posthole Pete protested *obstreperously* (imagine what that was like!).

He was so upset he couldn't think straight.

Could you defend him with cool logic and unwavering intelligence?

Problem 11

Weighty Evidence

On May 01, Club Fittest was broken into and the exercise equipment was battered. There were several baseball bats left strewn about. Damage was extensive. The time of the break-in was placed at 12:24, just after midnight.

In searching for clues, ace detective Alethia Athorn noticed that the computerized weigh scale was still turned on. Quickly and deftly she turned the printer on and printed out the most recent weights recorded in the computer:

Date	Time	Mass
May 01	8 45	52 kg
May 01	8 59	49 kg
May 01	12 21 15	75 kg
May 01	12 21 35	75 kg
May 01	12 22 10	70 kg

As Alethia was reading the print out, police hauled in three noisy females who were found laughing hilariously in the coffee shop next door. Here they are:

Wilma Whaley   Harriet Hustle   Sharon Swimmy

When confronted with the weigh scale printout, Harriet haughtily exclaimed, "How could I possibly weigh that much!" The others echoed Harriet's disdain. But none of them would stand on the scale to be weighed.

Detective Alethia asked to borrow Harriet's "Tail Piece" and before she could say no, placed it on the weigh scale. It registered 5 kg.

Do you think that any of the three were guilty?

Problem 12

Time Out for DeciDays

Security Investigator Manfred Metric SI, a brother in law of Magnum PI the famous private investigator, didn't like filling in the Date and Time everytime he made a note in his SI Note Pad. It took him too long, he said.

He thought he had a solution. He would use the metric prefixes — deci, centi, etc.

One day would be 24 hours.

Then a deciday would be 2.4 hours or, taking out his trusty calculator, 2 hours and 24 minutes.

He thought that would be quite good because he could say that in the morning he worked about a deciday. Other SI agents worked longer of course.

The other SI agents also thought the idea of decidays was just plain silly — "sillidays" — and they laughed themselves silly.

Manfred was not impressed.

He went about making himself a table, just to see how long his new time units would be.

Manfred's unit	Days	Equivalent to hours	minutes	seconds
day	1	24	00	00
deciday	0.1	2	24	00
centiday	0.01	0	24	00
milliday	0.001			
decaday	10			

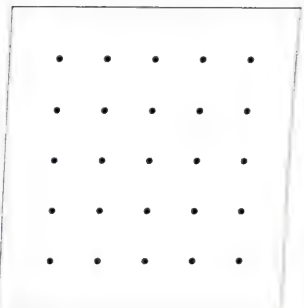Can you help Manfred complete his table (with calculator)?

Using these new units, how long is a class period?

How long is recess? Christmas vacation?

Problem 13

Capturing Pentagons on the Geoboard

Here is a pentagon captured on a geoboard:
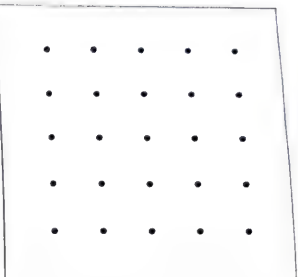

WANTED:
The Largest Pentagon on the Geoboard

- Capture
- Provide Measurements
  - area
  - perimeter

Problem 14

Capturing Hexagons on the Geoboard

Here is a Hexagon captured on a geoboard:

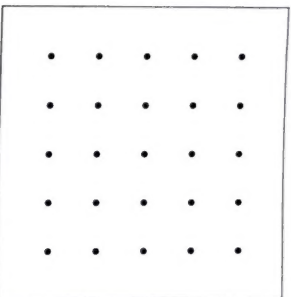## WANTED:
The Largest Hexagon on the Geoboard

- Capture
- Provide Measurements
  - area
  - perimeter

Problem 15

Capturing Octagons on the Geoboard

Here is a octagon captured on a geoboard:

WANTED:
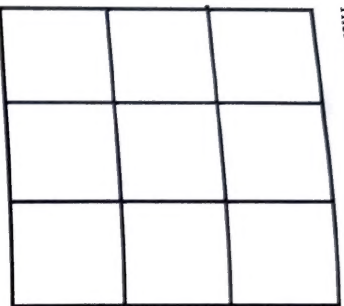## The Largest Octagon on the Geoboard

- Capture
- Provide Measurements
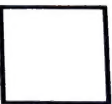 - area
  - perimeter

Problem 16

Search and Find

There are 14 different squares hiding in the net below:

There are 9 squares of this size:
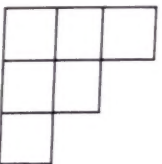
Can you search and find the rest?

Problem 17

Capture Through Measurements

These two culprits were recently captured through their measurements alone.

Culprit A   Culprit B

They were described as a "pair of square grid polygons" with measurements:

"A" perimeter 10 units and area 6 square units
"B" perimeter 12 units and area 6 square units

Still on the loose are two "square grid polygons" with measurements:

- perimeter 12 units and area 8 square units
- perimeter 14 units and area 8 square units

Can you make posters for these two?

Problem 18

Recording a Bath

After solving the case of the missing skunk, Detective Aromata was in desperate need of a bath. The story of what happened is told in the graph. Can you retell the story using ordinary English?

Volume in Litres (vertical axis): 0, 10, 20, 30, 40, 50, 60, 70, 80

Time (horizontal axis): 8:15, 8:16, 8:17, 8:18, 8:19, 8:20, 8:21, 8:22, 8:23, 8:24, 8:25, 8:26, 8:27